Design of Complex Sensor-Actuator-Systems (EKOSAS)

W. Dötzel, J. Mehner, F. Bennini

The goal of the project EKOSAS is to develop methods and tools for modeling and simulation of Micro-Electro-Mechanical-Systems (MEMS). Essential points are the coupling of different physical domains to the electronic circuitry in static and dynamic case. The design environment covers T-CAD for process simulations, FEM\BEM for coupled fields on the component level and VHDL-AMS for system level simulations. Finally, these methods shall be tested and optimised on a set of complex sensor-actuator-systems. In particular, the goals are:

- Development of tools for computer aided generation of reduced order macromodels for MEMS
- Development of interfaces for data exchange between etching simulation tool (SIMODE) and FEM (ANSYS) \BEM (CAPA) simulation tools.
- Integration of all submodels in one model for system simulations
- Benchmark tests (micromirror array, ultrasonic transmitters and receivers, position sensor, inclination sensor)

The subprojects of the Chemnitz University of Technology are focused on the automatic generation of reduced order macromodels for system simulations:

A common engineering approach to analyze complex systems is to approximate the unknowns by a series of weighted linearly independent shape functions:

$$u_i(t, x_i, y_i, z_i) \approx u_{eq} + \sum_{j=1}^{m} q_j(t) \cdot \phi_j(x_i, y_i, z_i) \qquad (1)$$

Such an approach is well known as Galerkin method where u_i are the time dependent nodal displacements of the FE-model and ϕ_j the linear shape functions which are scaled by time dependent factors q_j. Choosing eigenvectors of the linear system as shape functions is very efficient since they describe the real deformation state of the structure by a minimal number of functions with high accuracy. Furthermore, eigenvectors can be computed automatically by a modal analysis. In general, "Eq. (1)" describes a coordinate transformation of finite element displacement coordinates to modal coordinates of the macromodel. The deformation state of the structure given by n nodal displacements u_i ($i=1,2,\ldots,n$) is now represented by a linear combination of m modes weighted by their amplitudes q_j ($j=1,2,\ldots,m$) where $m << n$.
According to [1], one can use the second energy formulation of Lagrange to establish the governing equations of motion describing the ROM of an electrostatic actuated MEMS structures in modal coordinates:

$$m_j \ddot{q}_j + 2\xi_j \omega_j m_j \dot{q}_j + \frac{\partial W_{st}(q_1,\ldots,q_m)}{\partial q_j} = \frac{1}{2}\sum_r \frac{\partial C_{ks}(q_1,\ldots,q_m)}{\partial q_j} \cdot (V_k - V_s)^2 + \sum_{i=1}^{n} \phi_j^i \cdot F_i \qquad (2)$$

where m_j is the modal mass, ω_j the eigenfrequency, ξ_j the linear modal damping ratio, W_{st} the modal strain energy function, C_{ks} the modal capacity-stroke function, r the number of capacities which are involved for microsystems with multiple electrodes, V the applied electrode voltage and F_i a local force acting at the i-th node. Furthermore, to export the ROM to a network simulator one must include a voltage-current relationship. The current I at each electrode k is defined by:

$$I_k = \frac{\partial Q_k}{\partial t} = \sum_r \left(C_{ks} \cdot \left(\frac{\partial V_k}{\partial t} - \frac{\partial V_s}{\partial t} \right) + \frac{\partial C_{ks}}{\partial t} \cdot (V_k - V_s) \right) \qquad (3)$$

An essential prerequisite to establish "Eq. (2)" and "Eq. (3)" are proper modal strain energy and capacity-stroke functions. Both are derived from a series of FE runs at various deflections states in the operating range. The acquired data are fitted to polynomial functions in order to compute the local derivatives which describe force and stiffness terms [2]. This process is computationally expensive but has to be done just once. The result is a black-box model that can be applied to any load situation. However, it has been found that the distinction of dominant and relevant mode shapes speeds up this process considerably. Dominant modes are characterized by large amplitude. Their interactions to all other mode shapes, dominant and relevant, are regarded throughout. Relevant modes contribute to the final solution but do not affect among each other. Consequently the multivariable function fit can be reduced to a series of functions with a lower number of variables. It turned out that two dominant modes are sufficient for most applications. The polynomials can then by described by the following series representation:

$$P(q_1, q_2, q_3, q_4, \cdots, q_m) \approx P(q_1, q_2, q_3) + \sum_{j=4}^{m} P(q_1, q_2, q_j) - (m-3) \cdot P(q_1, q_2, 0) \tag{4}$$

This approximation yields to insignificant errors. Remarkable is that not only the number of polynomial coefficients can be reduced but also the number of sample points. For example, if the nonlinear strain energy is computed for five mode shapes and six modal amplitudes of each mode, the number of data points would be 6^5=7776 compared to 3×6^3=648 when two modes are classified as dominant and the three others as relevant. Furthermore, the fit is limited to functions with three variables, which allows one to use simple and fast algorithms like the well known least square fit. Depending on the FE model size and the number of mode shapes, which are includes in the ROM procedure, the data acquisition is typically an over-night job.
"Eq. (2)" and "Eq. (3)" describe reduced order models in modal coordinates, which affect the whole structural deformation. Under some circumstances where the structure undergoes temporary constraints in local coordinates (e.g.: contact problems), an interface is needed, which couples nodal displacements and modal amplitudes at specific points of the structure. A bi-directional coupling between both coordinates is done by means of Lagrangian multipliers λ_i, which represent internal forces due to deformation states whether in local or modal coordinate as follows:

$$\begin{bmatrix} K_{11}^{qq} & K_{12}^{qq} & K_{13}^{qq} & K_{11}^{qV} & K_{12}^{qV} & -\phi_1^{u1} & -\phi_1^{u2} & 0 & 0 \\ K_{21}^{qq} & K_{22}^{qq} & K_{23}^{qq} & K_{21}^{qV} & K_{22}^{qV} & -\phi_2^{u1} & -\phi_2^{u2} & 0 & 0 \\ K_{31}^{qq} & K_{32}^{qq} & K_{33}^{qq} & K_{31}^{qV} & K_{32}^{qV} & -\phi_3^{u1} & -\phi_3^{u2} & 0 & 0 \\ K_{11}^{Vq} & K_{12}^{Vq} & K_{13}^{Vq} & K_{11}^{VV} & K_{12}^{VV} & 0 & 0 & 0 & 0 \\ K_{21}^{Vq} & K_{22}^{Vq} & K_{23}^{Vq} & K_{21}^{VV} & K_{22}^{VV} & 0 & 0 & 0 & 0 \\ -\phi_1^{u1} & -\phi_2^{u1} & -\phi_3^{u1} & 0 & 0 & 0 & 0 & 1 & 0 \\ -\phi_1^{u1} & -\phi_2^{u2} & -\phi_3^{u3} & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & K^{u1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & K^{u2} \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ V_1 \\ V_2 \\ \lambda_1 \\ \lambda_2 \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ I_1 \\ I_2 \\ 0 \\ 0 \\ F_1 \\ F_2 \end{bmatrix}$$

Modal forces-Amplitudes relationship
Voltage-Current relationship
Lagrangian multiplieres
Nodal forces-Displacements relationship

Modal coordinate | Bidirectional coupling | Local coordinate

$$K_{ij}^{qq} = \frac{\partial^2 W_{st}}{\partial q_i \partial q_j} - \frac{1}{2} \sum_r \frac{\partial^2 C_r}{\partial q_i \partial q_j} \cdot V_r^2 \, ; \; K_{ij}^{qV} = \frac{\partial^2 W_{elek}}{\partial q_i \partial V_j} \, ; \; K_{ij}^{VV} = \frac{\partial I_i}{\partial V_j} \, ; \; K_{ij}^{Vq} = \frac{\partial I_i}{\partial q_j} \, . \tag{5}$$

This equation describes a ROM with three modal DOFs and two electrodes, which is coupled with two nodes from the corresponding FE-model. Fig. 1 shows an example of a contact problem of an electrostatically actuated one side clamped beam with contact pad, treated with reduced order modeling techniques. The upper plot shows the bending line of the beam where the contact is placed at the right end. In the lower part, the contact pad is situated at the center of the beam. It could be shown that only six linear mode shapes are able to map the deformation state of the structure before and after contact with an error of less than 1% in comparison with the full finite element analysis.

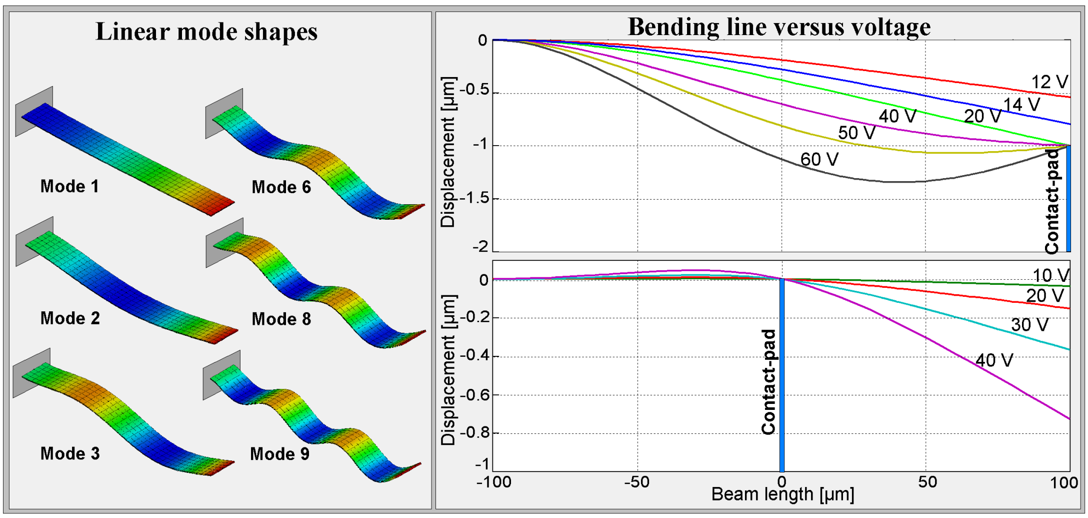

Fig. 1: Voltage vs. displacement functions of a one side clamped beam with contact pad

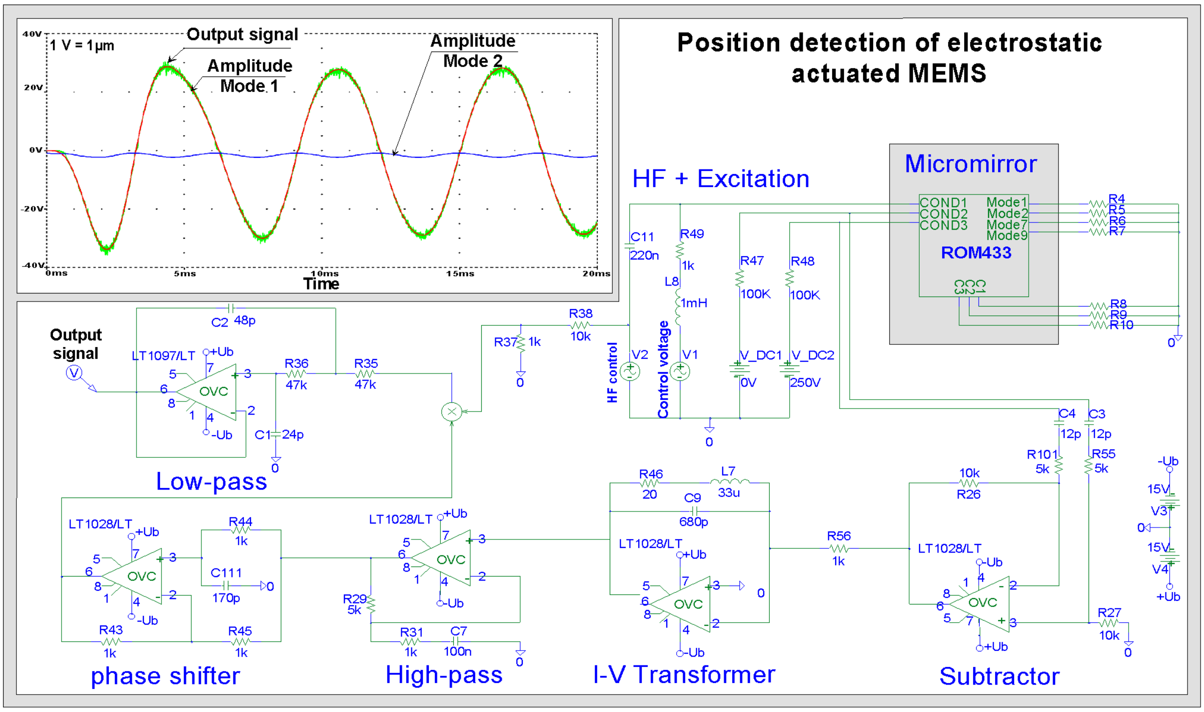

Fig. 2: System level simulation in PSPICE using a ROM of a micromirror

The ultimate goal of ROM is to obtain an accurate black-box model of the microsystem's behavior. Interface signals are limited to the voltage-current relationship at each electrode, essential inputs such as external loads (e.g. gravitation, pressure) and significant output quantities (e.g. a subset of displacements at characteristic points of the model). The black-box model can be exported and used for modeling systems as signal flow graphs (e.g. SIMULINK) or as networks (e.g. PSPICE). VHDL-AMS becomes more and more important in analyzing and simulating MEMS on the system level. It is much more flexible, because in addition to the across and through quantities (Network syntax) one can use further relations expressed by analytical terms inside the total system. Fig. 2 illustrates an example of a system simulation performed in PSPICE. It demonstrates a frequently used electronic circuitry to detect the position of electrostatic actuated microstructures. In this example, the ROM of a micromirror presented in [1] is used, where the current alteration on the ground electrodes is evaluated to obtain a proportional signal to the mirror plate position. Since the deflection state of the structure is decomposed in shape functions (Mode 1 the rotation, Mode 2 the translation; Mode7 and 9 the warp of the mirror plate), one can analyze the sensitivity of the circuit with respect to the mirror plate deformation and evaluate the errors.

References:

[1] F. Bennini, J. Mehner and W. Dötzel, *Computational Methods for Reduced Order Modeling of Coupled Domain Simulations*, 11 International Conference on Solid State Sensors and Actuators (Transducers 01), p. 260-263, Germany 2001

[3] J. E. Mehner, L.D. Gabbay and S.D. Senturia, *Computer-Aided Generation of Nonlinear Reduced-Order Dynamic Macromodels*, Journal of Microelectromechanical Systems, p. 270-277, June 2000