# Oscillations of the direct tunneling current in a MOS capacitor with ultra- thin gate dielectrics 

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#### Abstract

A novel quasi- classical expression is derived, that expresses the gate oxide thickness by the electric field strengths corresponding to two subsequent extrema in the leakage current oscillations. By analyzing various experimental gate current data, the new formula is successfully applied to determine the oxide thickness.


## 1. Introduction

Oscillations of the electron transmission through the triangular barrier has been firstly studied theoretically by Gundlach [1] in Metal-Insulator- Metal (MIM) structures. Although the leakage current oscillations have not been observed in MIM capacitors, experimental evidence toward the oscillations at high gate voltage has been revealed firstly by Maserjian et al. $[2,3]$ in Metal- Oxide- Semiconductor (MOS) structures with thin gate oxide.

To explain the electric field $(F)$ dependence of the measured tunneling current, the Fowler- Nordheim (FN) quasi- classical expression $J_{0}(F)$ for the tunnelling current is usually combined with the oscillatory amplitude $B$ of Gundlach's quantum- mechanical expression for the transmission coefficient, which is responsible for the oscillation of the total tunnelling current $J(F)$ :

$$
\begin{equation*}
J(F)=B J_{0}(F), \tag{1}
\end{equation*}
$$

where $\quad J_{0}(F)=A F^{2} \exp (-C / F) \quad$ and (2)
$B=\left[A i^{2}\left(-a L_{c b}\right)+(a / k)^{2} A i^{12}\left(-a L_{c b}\right)\right]^{-1}$.
A and C in the FN expression (2) are defined as:
$A=q^{3} m_{s i}^{*} /\left(16 \pi^{2} \hbar m_{o x}^{*} \phi_{B}\right)$
and

$$
C=2 \int_{0}^{\phi_{B}} k[\phi] d \phi=(8 \pi / 3 q \hbar)\left(2 m_{o x}^{*} \phi_{B}^{3}\right)^{1 / 2}
$$

where $\phi_{B}$ is the semiconductor- oxide barrier height, $m_{s i}^{*}$ and $m_{o x}^{*}$ are the electron effective masses in the silicon substrate and gate oxide, respectively. $A i(x)$ and $A i^{\prime}(x)$ in Eq.(3) are the Airy function and its derivative, respectively. The coefficient $a$ and the distance $L_{c b}$, travelled by the electron with energy E in the conduction band of $\mathrm{SiO}_{2}$ are given by $a=\left(2 m_{o x}^{*} q F / \hbar^{2}\right)^{1 / 3}$
and $L_{c b}=t_{o x}-\left(\phi_{B}-E\right) / q F$, respectively.
Analytical investigations of the oscillatory gate current shows that the positions of the extrema in the current oscillation depend on the internal characteristics of the system. These are the gate oxide thickness $t_{o x}$, the barrier height $\phi_{B}$, the effective mass and the Fermi energy of electrons $E_{F}$ in the system, and also on the strength of the applied electric field $F$. This fact allows us to express the oxide thickness via the values of the electric field strength F at the extrema of the oscillations $F=F_{n}$. The conventional method to determine the gate oxide thickness is based on the oscillative nature of the quantum prefactor B given by Eq.(3), which gets maximal values at zero points of the Airy function, $A i\left(-a L_{c b}\right)_{F=F_{n}}=0$, and minimal values at zero points of the derivative of the Airy function, $A i^{\prime}\left(-a L_{c b}\right)_{F=F_{n}}=0$.

We present a new quasi- classical method to determine the gate oxide thickness by measuring the electric fields corresponding to two subsequent extrema of oscillations [4]. Careful analyses of experimental data of the gate current oscillations, to determine the oxide thickness by applying our method as well as the conventional method, shows that accuracy of our method is higher than that of the conventional method.

## 2. Determination of the oxide thickness from the oscillation period

To determine the oxide thickness $t_{o x}$ by the conventional method [5], the following qualitative relation between $t_{o x}$ and the measured values of the electric field $F_{n}$ at the $n$th maximum (or $n$th minimum) is used:

$$
\begin{equation*}
\left(\frac{2 q m_{o x}^{*} F_{n}}{\hbar^{2}}\right)^{1 / 3}\left(t_{o x}-\frac{\phi_{B}-E_{F}}{q F_{n}}\right)=K_{n}, \tag{4}
\end{equation*}
$$

where $K_{n}$ is the $n$th zero of the Airy function (of the derivative of the Airy function).

We suggest a quasi- classic theory to determine the oxide thickness by knowing the positions of two subsequent extrema in the current oscillations. According to the quantum-

(a)

(b)

Fig.1: (a) Tunneling of a particle with energy $U_{0}<E<U_{1}$ through a quantum barrier of width $b$ with subsequent transmission over a barrier of width $a$. (b) Tilting of the conduction band edge of $\mathrm{SiO}_{2}$ under the gate voltage. The line $A L$ corresponds to the voltage, where a crossover from the WKB regime to the FN regime by increasing the gate voltage takes place. The lines $A M$ and $A N$ characterize the conduction band tilting under the voltages $V_{1}$ and $V_{2}$, corresponding to two subsequent extrema in the current oscillations.
mechanic the transmission over a potential well or a potential barrier of width $a$, the transmission coefficient $|T|^{2}$ reaches the extremal values at the points $k a=\pi(n-\alpha)$ $n=1,2,3, \ldots$, which correspond to maxima for
$\alpha=0$ and minima for $\alpha=1 / 2$. Since the depth of a potential well $-U_{0}$ or the height of a barrier $U_{0}$ is constant, the wavelength $k=\sqrt{2 m\left(E-\left|U_{0}\right|\right) / \hbar^{2}}$ of the transmitted particle with energy $E$ takes also constant values at every point of motion over the well or barrier. If the spatial distance between two subsequent maxima or minima is $\Delta a$, then $\Delta a k=\pi$.

To understand the nature of oscillations deeper, we have studied the transmission of a particle with energy $U_{0}<E<U_{1}$ through the more complicated structure, shown in Fig.1a analytically. Minimizing $|T|^{2}$ with respect to $a$ gives the following condition for the extremal points:

$$
\begin{equation*}
\tan (2 k a)=\frac{2 k \kappa\left(q^{2}+\kappa^{2}\right) \tanh (\kappa b)}{\kappa^{2}\left(k^{2}-q^{2}\right)+\left(k^{2} q^{2}-\kappa^{4}\right) \tanh ^{2}(\kappa b)} \tag{5}
\end{equation*}
$$

where $q^{2}=2 m E / \hbar^{2}, \kappa^{2}=2 m\left(U_{1}-E\right) / \hbar^{2}$.
The transmission coefficient $|T|^{2}$ can be seen from Eq.(5) to take maximal and minimal values at $k a=\pi(n-\alpha)+\delta \alpha / 2 \quad$ for $\quad \alpha=0 \quad$ and $\alpha=1 / 2$, correspondingly. The shift $\delta \alpha$ occurs due to tunnelling of an electron through the potential barrier of width $b . \delta \alpha$ vanishes as $b \rightarrow 0$ or $U_{1} \rightarrow U_{0}$, and it does not depend neither on the distance $a$ nor on the electric field. The distance $\Delta a$ obeys again the relation of $\Delta a k=\pi$.

The wave number of an electron, which propagates in the tilted conduction band of silicon oxide, depends on the spatial coordinates, instead of $k=$ const as in the previous examples of electron motion over a potential barrier or well. Fig. 1 b shows the conduction band tilting of silica for different voltages. Imagine that an electron moves the distance $\left(t_{o x}-z_{n}\right)$ over the conduction band of $\mathrm{SiO}_{2}$ under the applied voltage $V_{n}$, which corresponds to the $n$th maximum or minimum. Dividing the distance of $\left(t_{o x}-z_{n}\right)$ to $N$ equal portions with length of $\Delta z_{i}$ around the point $z_{i}$, an electron wave number $k\left(z_{i}\right)=\sqrt{2 m_{o x}^{*}\left(E-U\left(z_{i}\right)\right) / \hbar^{2}}$ can be roughly considered to be constant moving over a
rectangular barrier of width $\Delta z_{i}$. Summing up over all portions and taking $N \rightarrow \infty$ we get the condition to determine the position of $n$th maximum for $\alpha=0$ or $n$th minimum for $\alpha=1 / 2$ :
$\int_{z_{n}}^{t_{o x}} \sqrt{\frac{2 m_{o x}^{*}}{\hbar^{2}}(E-U(z))} d z=\pi(n-\alpha)+\delta \alpha$,
where $\delta \alpha$ is a phase shift, $U(z)=\phi_{B}-q F z$. The distance $z_{1}$, e.g. for the first maximum or minimum can be found in Fig.1b from the triangle ACM as $z_{1}\left(\phi_{B}-E\right) / q F_{1}$. Routine calculations give the final formula for the oxide thickness $t_{o x}$ by the values of the electric fields at two subsequent maxima or minima $F_{1}$ and $F_{2}$ :

$$
\begin{align*}
& 3 \sqrt{2 m_{o x}^{*}} / 2 \hbar\left\{\left(q F_{2} t_{o x}-\phi_{B}+E\right)^{3 / 2} / q F_{2}-\right.  \tag{7}\\
& \left.\left(q F_{1} t_{o x}-\phi_{B}+E\right)^{3 / 2} / q F_{1}\right\}=\pi
\end{align*}
$$

To check the validity of our quasiclassical oxide thickness determination method, we utilize experimental data for two subsequent maxima or minima of the gate voltage $\left(V_{\max 1}, V_{\max 2} \quad\right.$ or $\left.\quad V_{\min 1}, V_{\min 2}\right)$ and the corresponding electric fields $\left(F_{\max 1}, F_{\max 2}\right.$ or $F_{\min 1}, F_{\min 2}$ ) from three different measurements $[2,6,7]$. In the estimations we used $\phi_{B}=3.1 \mathrm{eV}$ for the oxide barrier height and $E=E_{F}=0.275 \mathrm{eV}$ for the electron energy that has been obtained in [8] from self- consistent solutions of the Schrödinger- Poisson system for the MOS capacitor. The effective masses of electrons in the Si substrate and in the gate oxide are taken to be
$m_{s i}^{*}=0.916 m_{0} \quad$ and $\quad m_{o x}^{*}=0.51 m_{0}$, respectively, where $m_{0}$ is the free electron mass. The comparative estimations of the oxide thicknesses by applying our method and the conventional method are presented in Table I. The oxide thickness determined by our method differ from experimental results less than $5 \%$. Errors in the oxide thicknesses determined by the conventional method reach up to $90 \%$.

## 3. Conclusions

The tunnelling current oscillations provide a valuable tool to determine the thickness and to check the quality of ultrathin gate oxides. We suggest a novel quasi- classical

Table I:Estimations of the oxide thickness from the current oscillations data of Refs. [2, 6, 7].

| $\begin{aligned} & V_{\text {max }}\left(V_{\text {min }}\right) \\ & \text { (in Volt) or } \\ & F_{\text {max }}\left(F_{\text {min }}\right. \text { ) } \\ & \text { (in MV/cm) } \end{aligned}$ | Exper. <br> Values <br> $t_{o x}, \mathrm{~nm}$ | Our Method <br> $t_{o x}, \mathrm{~nm}$ <br> (Error,\%) | Conven. <br> Method <br> $t_{o x}, \mathrm{~nm}$ <br> (Error,\%) |
| :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Ref.[2] } \\ & V_{\min 1}=5.15 \end{aligned}$ | 4.5 | 4.47 (0.6) | 0.4 (91) |
| $\begin{aligned} V_{\min 2} & =6.12 \\ V_{\max 1} & =5.58 \\ V_{\max 2} & =5.58 \end{aligned}$ |  | $4.73(5.0)$ | $1.2(73)$ |
| $\begin{aligned} & \text { Ref.[6] } \\ & V_{\min 1}=5.05 \end{aligned}$ | 4.5 | 4.52 (0.4) | 0.44 (90) |
| $\begin{aligned} V_{\min 2} & =6.00 \\ V_{\max 1} & =5.50 \\ V_{\max 2} & =6.47 \end{aligned}$ | 4.5 | $4.60(2.0)$ | $1.24(72)$ |
| $\begin{aligned} & \text { Ref.[7] } \\ & F_{\min 1}=7.9 \end{aligned}$ | 4.2 | 4.24 (1.0) | 2.8 (33) |
| $\begin{aligned} & F_{\min 2}=10.2 \\ & F_{\max 1}=7.4 \\ & F_{\max 2}=8.8 \end{aligned}$ | 5.2 | $5.30(2.0)$ | $3.7 \text { (28) }$ |

method to find the oxide thickness. Our expressions (6) and (7) depend, apart from the electric field, also on the effective mass of an electron in the oxide and on the barrier height, which allow us to determine $m_{o x}^{*}$ and $\phi_{B}$ for a sample with a given thickness.

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